soaps, but the particles of colloidal cetyl sulphonic acid are very much more prominent.

In conclusion, we desire to express our thanks to the Research Fund of the Chemical Society and to the Colston Society of the University of Bristol for grants with which the pure materials were purchased.

DESCRIPTION OF PLATES 10 AND 11.

Figs. 1 and 5, 0·6 N Na oleate; fig. 2, 0·2 N Na palmitate; fig. 3, 1·0 N Na laurate; fig. 4, 0·5 N Na stearate; figs. 6, 7, 8, and 9, 0·5 N K stearate; figs. 10 and 11, 0·5 N cetyl sulphonic acid. Figs. 2–11 refer to curds. Magnification $\times 500$ in figs. 1, 2, 5, 7, 8, and 11; $\times 600$ in figs. 3, 4, 6, 9, and 10.

The Transmission of Electric Waves around the Earth's Surface.

By H. M. Macdonald, F.R.S.

(Received January 10, 1921.)

In the 'Proceedings,'* Prof. G. N. Watson discusses the effect of a perfectly conducting layer in the atmosphere at a uniform height above the earth's surface on the transmission of electric waves round the earth. The mathematical treatment adopted by him assumes that the time factor $e^{-\iota k C t}$ can be removed from all the equations, and that the analytical results thus obtained represent the effect of a simple oscillator placed near the earth's surface. The assumption that the time factor can be removed is equivalent to assuming that a steady state of oscillations exists in the space between the two spheres, such that at the end of a period the amplitudes of the electric and magnetic forces are identical at each point with the values they had at the beginning of the period. Now, when the surfaces of both spheres are perfectly conducting, no energy is transmitted across either surface, and therefore, if there is a steady state of oscillations in the space, the total energy in the space must be constant. When there is an oscillator in the space emitting electric waves, there is a finite amount of energy radiated from the oscillator in each period, and therefore the total energy in the space does not remain constant; it follows that in such a case there is no steady state of

* 'Roy. Soc. Proc.,' A, vol. 95, p. 546.

oscillations and the mathematical problem involved cannot be treated by assuming that there is a time factor which can be removed.

The effect of any electric disturbance set up in the space between the two perfectly conducting spherical surfaces can be expressed in the usual way in terms of the natural periods of the space. If a and b are the radii of the two concentric spherical surfaces, the equation which determines the periods corresponding to the Legendre function of order n, when the disturbance is such that the lines of magnetic force are circles with the axis of the harmonics as a common axis, is

$$\frac{d}{da}\{a^{\frac{1}{2}}\mathrm{J}_{n+\frac{1}{2}}(\kappa a)\}\frac{d}{db}\{a^{\frac{1}{2}}\mathrm{K}_{n+\frac{1}{2}}(\iota\kappa b)\} - \frac{d}{db}\{b^{\frac{1}{2}}\mathrm{J}_{n+\frac{1}{2}}(\kappa b)\}\frac{d}{da}\{a^{\frac{1}{2}}\mathrm{K}_{n+\frac{1}{2}}(\iota\kappa a)\} = 0, \quad (1)$$

where $\kappa\mathrm{V}$ is the frequency.* When the disturbance consists of a single pulse at a point near the surface of the inner sphere for which $\theta = 0$, it can be shown that the value of γ, the magnetic force at any time, is given by the relation

$$\begin{aligned}\psi = \gamma r\sin\theta = \mathrm{A}\sum_{n=1}^{\infty}\sum_{\kappa}(n+\tfrac{1}{2})(vu_a'-uv_a')\\ (1-\mu^2)\frac{d\mathrm{P}_n}{d\mu}\cos\kappa\mathrm{V}t \Big/ [a\{1-n(n+1)/\kappa^2a^2\}-b\{1-n(n+1)/\kappa^2b^2\}(v_a'/v_b')^2]\\ +\mathrm{B}\sum_{n=1}^{\infty}\sum_{\kappa}(n+\tfrac{1}{2})(vu_a'-uv_a')\\ (1-\mu^2)\frac{d\mathrm{P}_n}{d\mu}\sin\kappa\mathrm{V}t \Big/ [\kappa a\{1-n(n+1)/\kappa^2a^2\}-\kappa b\{1-n(n+1)/\kappa^2b^2\}(v_a'/v_b')^2],\end{aligned} \quad (2)$$

where $v = (-)^n(\pi/2)^{\frac{1}{2}}(\kappa r)^{\frac{1}{2}}\mathrm{J}_{-n-\frac{1}{2}}(\kappa r)$, $u = (\pi/2)^{\frac{1}{2}}(\kappa r)^{\frac{1}{2}}\mathrm{J}_{n+\frac{1}{2}}(\kappa r)$, and the summation with respect to κ extends to all the values of κ that satisfy equation (1), that is the equation

$$v_b'u_a'-v_a'u_b' = 0. \quad (1')$$

The effect due to any symmetrical disturbance can be deduced from the relation (2).

When the inner sphere is imperfectly conducting, or when the space outside the outer sphere is imperfectly conducting, or both are imperfectly conducting, a similar, though necessarily somewhat more complicated, analysis applies. For example, when the inner sphere is imperfectly conducting, the outer surface, radius b, being perfectly conducting, the equation which deter-

* *Cf.* J. J. Thomson, 'Recent Researches in Electricity and Magnetism,' § 315 (1893)

mines the frequencies and rates of decay corresponding to the Legendre function of order n is

$$\frac{\partial}{\partial a}\{a^{\frac{1}{2}}J_{n+\frac{1}{2}}(\kappa a)\}\frac{\partial}{\partial b}\{b^{\frac{1}{2}}K_{n+\frac{1}{2}}(\iota\kappa b)\}-\frac{\partial}{\partial a}\{a^{\frac{1}{2}}K_{n+\frac{1}{2}}(\iota\kappa a)\}\frac{\partial}{\partial b}\{b^{\frac{1}{2}}J_{n+\frac{1}{2}}(\kappa b)\}$$

$$=\frac{\iota\kappa\sigma}{4\pi V}\left[J_{n+\frac{1}{2}}(\kappa a)\frac{\partial}{\partial b}\{b^{\frac{1}{2}}K_{n+\frac{1}{2}}(\iota\kappa b)\}-K_{n+\frac{1}{2}}(\iota\kappa a)\frac{\partial}{\partial b}\{b^{\frac{1}{2}}J_{n+\frac{1}{2}}(\kappa b)\}\right]$$

$$\frac{\partial}{\partial a}\{a^{\frac{1}{2}}J_{n+\frac{1}{2}}(\kappa' a)\}\Big/ J_{n+\frac{1}{2}}(\kappa' a),$$

where $\kappa'^2 = -4\pi\iota\kappa V/\sigma$, and the values of κ are complex quantities. It can be readily shown that when the conductivity of the inner sphere is of the same order as the conductivity of sea-water, the rates of decay associated with the frequencies that are of the order of those of the waves of wireless telegraphy are small when $b-a$ is small compared with b; it follows that the time required to establish an approximately steady state is impracticably long.*

A Comparison of Magnetic Declination Changes at British Observatories.

By C. Chree, Sc.D., LL.D, F.R.S.

(Received January 27, 1921.)

In spite of the very varied use of the compass needle for purposes of navigation and surveying, there is little really known about the closeness in the parallelism of daily magnetic changes in different parts of the United Kingdom. The mean annual values from magnetic observatories show that, since 1910, secular change of declination has been at least approximately the same throughout England, the south of Scotland, and the south-west of Ireland. Thus the conditions have been favourable for an enquiry into the parallelism of other changes.

Diurnal inequalities from five selected quiet days a month were published for Falmouth as well as Kew up to 1912. A comparison of results from a number of years combined had shown little difference between the diurnal inequalities at the two stations as regards the range. Difference in local

* The rates of decay of electrical currents in an imperfectly conducting sphere have been investigated by Prof. H. Lamb, "Electrical Motions in Spherical Conductors," 'Phil. Trans.,' Part II, p. 530 (1883).